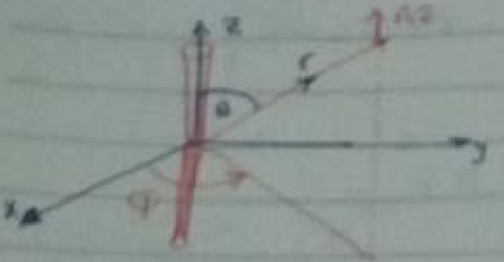
١٣/٨ lec

$$A(r) = A_z = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r} \hat{z}$$

$\hookrightarrow$ Cartizian Coordinates (x, y, z)

We Must charge These Coordinates into spherical Coordinate

$(x, y, z) \rightarrow (r, \theta, \phi)$



$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_x = 0 \quad , A_y = 0 \quad A_z = \ldots$

$A_r = A_z \cos\theta \; \hat{r} \quad \rightarrow A_r = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r} \cos\theta \; \hat{r}$

$A_\theta = -A_z \sin\theta \; \hat{\theta} \quad \rightarrow A_\theta = \frac{-\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r} \sin\theta \; \hat{\theta}$

$A_\phi = 0$

* Find $\bar{H}$ for infinitismal dipole

$$\bar{H} = \frac{1}{\mu}(\nabla \times \bar{A})$$

$$\bar{H} = \frac{1}{\mu} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \cancel{\frac{\partial}{\partial \phi}} \; 0 \\ A_r & rA_\theta & \cancel{r\sin\theta A_\phi} \end{vmatrix}$$

$\downarrow$

$\left.\begin{matrix} H_r \\ H_\theta \\ H_\phi \end{matrix}\right\}$???

$\ast\ A_\phi = 0 \Rightarrow \frac{\partial}{\partial\phi} = 0$

① $\ast\ H_r = 0$

② $H_\theta = 0$

③ $H_\phi = \frac{1}{\mu}\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right] \ast r\sin\theta\ \hat{\phi}$

$$\frac{\partial}{\partial r}(rA_\theta) = \frac{\partial}{\partial r}\left[\cancel{r}\frac{\mu}{4\pi} I_o\Delta l \frac{e^{-JBr}}{\cancel{r}}\sin\theta\right]$$

$$= \frac{JB\mu}{4\pi} I_o\Delta l\, e^{-JBr}\sin\theta$$

$$\frac{\partial}{\partial\theta}(A_r) = \frac{\partial}{\partial\theta}\left[\frac{\mu}{4\pi} I_o\Delta l \frac{e^{-JBr}}{r}\cos\theta\right]$$

$$= \frac{-\mu}{4\pi} I_o\Delta l \frac{e^{-JBr}}{r}\sin\theta$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{JB\mu}{4\pi} I_o\Delta l\, e^{-JBr}\sin\theta + \frac{\mu}{4\pi} I_o\Delta l \frac{e^{-JBr}}{r}\sin\theta\right]\hat{\phi}$$

$$= \frac{1}{\cancel{\mu} r}\frac{\cancel{\mu}}{4\pi} I_o\Delta l\, e^{-JBr}\sin\theta\left[JB + \frac{1}{r}\right]\hat{\phi} \ast \frac{B^2}{B^2}$$

$\frac{B^2}{B^2}$ ادخل r جوا

$$\boxed{H_\phi = \frac{I_o\Delta l}{4\pi} e^{-JBr}\sin\theta\, B^2\left[\frac{J}{Br} + \frac{1}{B^2r^2}\right]\hat{\phi}}\ \# \Rightarrow ①$$

* لو جه في الامتحان هيجيب H_ϕ فقط دليين A_ϕ لازم نعوض بيه

Find $\bar{E}$ in infinitismal dipole

الدا

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow \frac{\partial}{\partial t} = J\omega$$

$$\boxed{\nabla \times \bar{H} = J\omega \epsilon \bar{E}}$$

$$\bar{E} = \frac{1}{J\omega e}\left[\nabla \times \bar{H}\right]$$

$$E = \frac{1}{J\omega e}\,\frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ \cancel{Hr} & r\cancel{H\theta} & r\sin\theta\, H\varphi \end{vmatrix}$$

$$Er = \frac{\partial}{\partial \theta}\left[r\sin\theta\, H\varphi\right]\hat{r}$$

$$Er = \frac{\partial}{\partial \theta}\left[r\sin\theta\,\frac{I_o\,\Delta l}{4\pi}\,e^{-JBr}\sin\theta\, B^2\left[\frac{J}{Br}+\frac{1}{B^2r^2}\right]\right]\hat{r} * \frac{1}{r^2\sin\theta}$$

$\sin^2\theta$

$* \frac{1}{J\omega e}$

$\frac{\partial}{\partial\theta} = 2\sin\theta\cos\theta$

$$* Er = \left[\frac{I_o\,\Delta l}{4\pi}\,\cancel{r}\,\frac{e^{-JBr}}{\theta}\,B^2\, 2\sin\theta\cos\theta\left[\frac{J}{B\cancel{r}}+\frac{1}{B^2r^2}\right]\hat{r}\right] * \frac{1}{r^2\sin\theta} * \frac{1}{J\omega e}$$

$$Er = \frac{1}{J\omega\epsilon}\,\frac{2 I_o\,\Delta l}{4\pi}\,B^2\,e^{-JBr}\left[\frac{J}{B}+\frac{1}{B^2 r}\right]\cos\theta\,\frac{1}{r^2}\,\hat{r}$$

$$\omega\mu = \eta B \qquad , \eta = \qquad B = \omega\sqrt{\mu\epsilon}$$

* Finaly

① $H\phi = \frac{I_0 \Delta l}{4\pi} B^2 e^{-JBr} \left[\frac{J}{Br} + \cancel{\frac{1}{B^2 r^2}} \right] \sin\theta \, \hat{\phi}$ #

[2] $Er = \frac{I_0 \Delta l}{4\pi} \left(2 \omega \mu B \right) e^{-JBr} \left[\cancel{\frac{1}{B^2 r^2}} + \cancel{\frac{1}{J B^3 r^3}} \right] \cos\theta \, \hat{r}$

[3] $E\theta = \frac{I_0 \Delta l}{4\pi} (\omega \mu B) e^{-JBr} \left[\frac{J}{Br} + \cancel{\frac{1}{B^2 r^2}} + \cancel{\frac{1}{J B^3 r^3}} \right] \sin\theta \, \hat{\theta}$

[4] $E\phi = 0$

⑤ $Hr = 0$

[6] $H\theta = 0$

$$\boxed{I(z) = I_0 \, \mathrm{rect}\left(\frac{z}{\Delta l} \right)}$$

* for far field approximation

$$\boxed{\frac{1}{r^2} = 0} , \quad \boxed{\frac{1}{r^3} = 0}$$

* ① $I(z) = I_0 \, \mathrm{rect}\left(\frac{z}{\Delta l} \right)$

[2] $H\phi = \frac{I_0 \Delta l}{4\pi} B^2 e^{-JBr} \sin\theta \frac{J}{Br}$

$$\boxed{H\phi = JB \frac{I_0 \Delta l}{4\pi} \frac{e^{-JBr}}{r} \sin\theta \, \hat{\phi}}$$

* ② $H_r = H\theta = 0$

③ $Er = 0$

(cc)

4 $E_\theta = \frac{I_o \Delta l}{4\pi} \omega \mu B e^{-JBr} * \frac{J}{Br} \sin\theta \,\hat{\theta}$ //

$$\boxed{E_\theta = J\omega\mu \,\frac{I_o \Delta l}{4\pi} \,\frac{e^{-JBr}}{r} \,\sin\theta \,\hat{\theta}}$$ #

*

① given $I(z)$

[2] $A_z = \frac{\mu}{4\pi} \frac{e^{-JBr}}{r} F.T\left[I(z) \right]$

$$\boxed{A_z = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-JBr}}{r}}$$

[3] $\boxed{E_\theta = J\omega A_z \sin\theta}$

$\frac{B}{\omega\mu} = \frac{1}{\eta}$

$\eta = \frac{\omega\mu}{B}$

[4] $\boxed{H_\varphi = \frac{1}{\eta} E_\theta} \Rightarrow \frac{H_\varphi}{E_\theta} = \frac{1}{\eta}$

I

σ_v

حفظ لحل المسائل

* Antenna Parameters

لجد الحصول على E و $\bar{H}$

① Poynting Vector $\bar{P}$

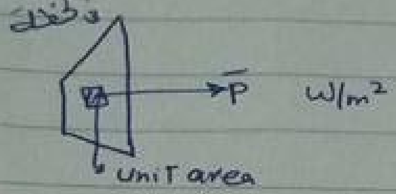
$\bar{P} = \bar{E} \times \bar{H}$ W/m^2

مكان ال
power

V I

is The instantaneous Power flow Per unit area)

تعريف هو متوسط الطاقة التي عبرت خلال وحدة المساحة



[2] Average radiated Power density $\bar{P}_{av}$

$$P_{av} = \frac{1}{2} R_e [\bar{E} \times H^*] \qquad W/m^2$$

(is the $\boxed{\text{Time averaged}}$ Power flow per unit area)

المتوسط خلال فترة زمنية